解答

一、多選題：

1.	2.
DE	AB

二、填充題：

1.	2.	3.	4.(1)
$\frac{-k}{\sqrt{1-k^2}}$	$135°46'$	$120°$	$\frac{\sqrt{15}}{4}$
4.(2)	5.	6.	7.
4	24	$\frac{3}{7}$	$\sqrt{33}$
8.	9.(1)	9.(2)	10.
4	$\pm\frac{\sqrt{7}}{4}$	$-\frac{5}{4}$	$\frac{8\sqrt{7}}{7}$
11.(1)	11.(2)	12.	
$\frac{2\sqrt{6}}{7}$	$\frac{55}{29}$	74	

1004 高二數學 第一章 (國立彰化女中 105 學年度第一學期第一次段考高二數學試題)

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解析

一、多選題：

1. (A) $\sin1000° = \sin(720° + 280°) = \sin280°$，∵$280°$為第四象限角，∴$\sin1000° < 0$．

(B) $\cos(-270°) = \cos270° = 0$．

(C) $\cos80°\cos70° - \sin80°\sin70° = \cos(80° + 70°) = \cos150° = -\frac{\sqrt{3}}{2}$．

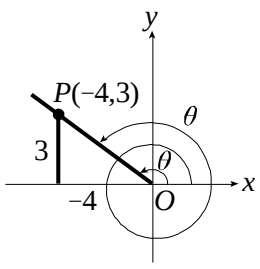
(D) $1 - 2\cos^2 50° = \sin100°$，∵$100°$為第二象限角，∴$\sin100° > 0$．

(E) $4\sin^3 70° - 3\sin70° = -(3\sin70° - 4\sin^3 70°) = -\sin210° = -\sin(180° + 30°)$

$= \sin30° = \frac{1}{2} > 0$．

故選(D)(E)．

2.



(A) $\cos\theta = -\frac{4}{5}$．

(B) $\cos2\theta = \cos^2\theta - \sin^2\theta = (-\frac{4}{5})^2 - (\frac{3}{5})^2 = \frac{7}{25}$．

(C) $\cos\frac{\theta}{2} = \pm\sqrt{\frac{1+\cos\theta}{2}} = \pm\sqrt{\frac{1+(-\frac{4}{5})}{2}} = \pm\frac{\sqrt{10}}{10}$．

(D) $\cos(90° + \theta) = -\sin\theta = -\frac{3}{5}$．

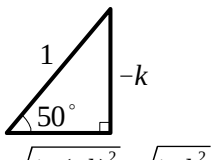
(E) $\cos(45° + \theta) = \cos45°\cos\theta - \sin45°\sin\theta = \frac{\sqrt{2}}{2} \times (-\frac{4}{5}) - \frac{\sqrt{2}}{2} \times (\frac{3}{5}) = -\frac{7\sqrt{2}}{10}$．

故選(A)(B)．

二、填充題：

1. ∵$\sin(-50°) = -\sin50° = k \Rightarrow \sin50° = -k$，如圖

∴$\tan230° = \tan(180° + 50°) = \tan50° = \frac{-k}{\sqrt{1-k^2}}$．



$\sqrt{1-(-k)^2} = \sqrt{1-k^2}$

2. 令 $\sin x = 0.7165$

由內插法

θ	$\sin\theta$
$45°40'$	0.7153
x	0.7165
$45°50'$	0.7173

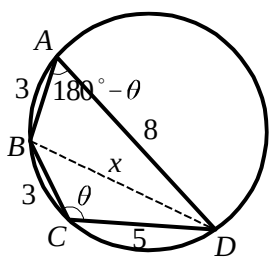
$$\frac{x-45°40'}{45°50'-45°40'}=\frac{0.7165-0.7153}{0.7173-0.7153}=\frac{12}{20}=\frac{3}{5}$$

$\Rightarrow x = 45°46' \Rightarrow \sin(45°46') = 0.7165$

$\Rightarrow \cos(90° + 45°46') = -\sin 45°46' = -0.7165$

$\therefore \theta = 135°46'$.

3.



如圖連 $\overline{BD}$，設 $\overline{BD} = x$

由 $\cos\theta + \cos(180° - \theta) = 0 \Rightarrow \frac{3^2+5^2-x^2}{2\times3\times5}+\frac{3^2+8^2-x^2}{2\times3\times8}=0 \Rightarrow x = 7$

$\therefore \cos\theta = \frac{3^2+5^2-7^2}{2\times3\times5}=-\frac{1}{2}$，$\therefore \theta = 120°$.

4. (1) $\because \sin A : \sin B : \sin C = a : b : c = 2 : 3 : 4$.

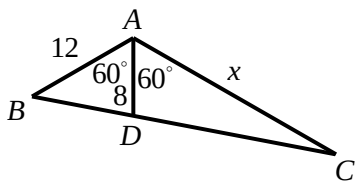
令 $a = 2r$，$b = 3r$，$c = 4r$

$\therefore \cos C = \frac{a^2+b^2-c^2}{2ab}=\frac{(2r)^2+(3r)^2-(4r)^2}{2(2r)(3r)}=-\frac{1}{4} \Rightarrow \sin C = \sqrt{1-\cos^2 C}=\frac{\sqrt{15}}{4}$.

(2) $\because \frac{1}{2}ab\sin C = \frac{4\sqrt{15}}{3}$

$\Rightarrow \frac{1}{2}\times 2r\times 3r\times\frac{\sqrt{15}}{4}=\frac{4\sqrt{15}}{3} \Rightarrow r=\frac{4}{3} \Rightarrow b = 3r = 4$.

5.



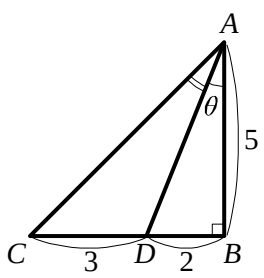
設 $\overline{AC} = x$，由面積$\triangle ABC$ = 面積$\triangle ABD$ + 面積$\triangle ACD$

得 $\frac{1}{2}\times12\times x\times\sin120° = \frac{1}{2}\times12\times8\times\sin60° + \frac{1}{2}\times8\times x\times\sin60°$

$\Rightarrow \frac{1}{2}\times12\times x\times\frac{\sqrt{3}}{2}=\frac{1}{2}\times12\times8\times\frac{\sqrt{3}}{2}+\frac{1}{2}\times8\times x\times\frac{\sqrt{3}}{2}$

$\Rightarrow 4x = 96 \Rightarrow x = 24$.

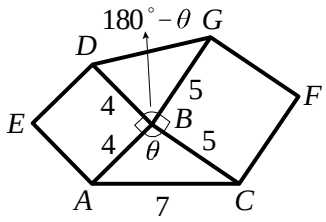
6.



令 $\overline{CD} = 3r$，$\overline{BD} = 2r$，則 $\overline{AB} = 5r \Rightarrow \tan\theta = \frac{2}{5}$

$\therefore \tan(\angle DAC)=\tan(45°-\theta)=\dfrac{\tan 45°-\tan\theta}{1+\tan 45°\times\tan\theta}=\dfrac{1-\frac{2}{5}}{1+\frac{2}{5}}=\dfrac{3}{7}$ ．

7.

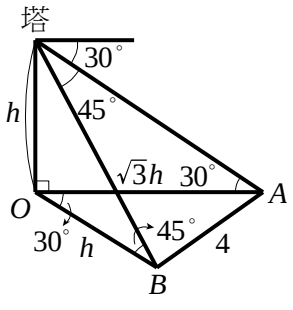


令$\angle ABC=\theta\Rightarrow\angle DBG=180°-\theta$

且$\cos\theta=\dfrac{4^2+5^2-7^2}{2\times4\times5}=-\dfrac{1}{5}\Rightarrow\cos(180°-\theta)=-\cos\theta=\dfrac{1}{5}$

$\therefore \overline{DG}=\sqrt{4^2+5^2-2\times4\times5\times\dfrac{1}{5}}=\sqrt{33}$ ．

8. 塔



如圖設塔底$O\Rightarrow\begin{cases}\overline{OA}=\sqrt{3}h\\ \overline{OB}=h\end{cases}$

由餘弦定理：$4^2=(\sqrt{3}h)^2+h^2-2\times\sqrt{3}h\times h\times\cos30°\Rightarrow16=h^2\Rightarrow h=4$ ．

9. (1) $\because(\sin x-\cos x)^2=(\dfrac{1}{2})^2\Rightarrow1-2\sin x\cos x=1-\sin 2x=\dfrac{1}{4}\Rightarrow\sin 2x=\dfrac{3}{4}$

$\Rightarrow\cos 2x=\pm\sqrt{1-\sin^2 2x}=\pm\dfrac{\sqrt{7}}{4}$ ．

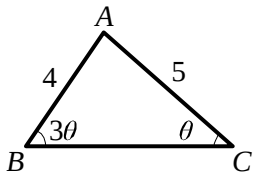
(2) $\sin3x+\cos3x=(3\sin x-4\sin^3x)+(4\cos^3x-3\cos x)$

$=3(\sin x-\cos x)-4(\sin^3x-\cos^3x)$

$=3(\sin x-\cos x)-4(\sin x-\cos x)(\sin^2x+\sin x\cos x+\cos^2x)$

$=3(\dfrac{1}{2})-4(\dfrac{1}{2})(1+\dfrac{3}{8})=-\dfrac{5}{4}$ ．

10.



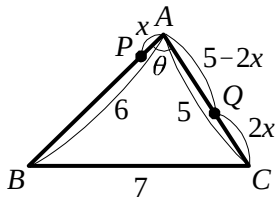
設$\angle C=\theta$，則$\angle B=3\theta$

由正弦定理：$\dfrac{5}{\sin3\theta}=\dfrac{4}{\sin\theta}\Rightarrow4\sin3\theta=5\sin\theta$

$\Rightarrow4(3\sin\theta-4\sin^3\theta)=5\sin\theta\Rightarrow\sin^2\theta=\dfrac{7}{16}\Rightarrow\sin\theta=\dfrac{\pm\sqrt{7}}{4}$（負不合）

$\therefore 2R=\dfrac{4}{\frac{\sqrt{7}}{4}}=\dfrac{16}{\sqrt{7}}\Rightarrow R=\dfrac{8}{\sqrt{7}}=\dfrac{8\sqrt{7}}{7}$ ．

11.



(1) 由餘弦定理：$\overline{BC}=\sqrt{6^2+5^2-2\times6\times5\times\cos A}=7$

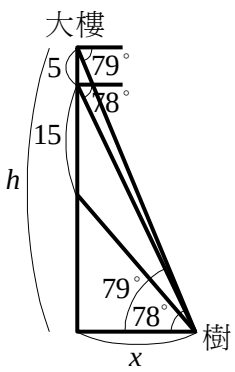
$$\therefore \cos B=\frac{6^2+7^2-5^2}{2\times 6\times 7}=\frac{5}{7}\Rightarrow \sin B=\frac{2\sqrt{6}}{7}\ .$$

(2) 令 $\overline{AP}=x\Rightarrow \overline{CQ}=2x\Rightarrow \overline{AQ}=5-2x$

$$\therefore \overline{PQ}=\sqrt{x^2+(5-2x)^2-2x(5-2x)(\frac{1}{5})}=\sqrt{\frac{29}{5}x^2-22x+25}$$

$\therefore$ 當 $x=-\frac{b}{2a}=\frac{55}{29}$ 時有最小值．

12.



設樹底離大樓 x 公尺，且樓高 h 公尺

$$\Rightarrow\begin{cases}\frac{h}{x}=\tan 79^\circ=5.145\\ \frac{h-5}{x}=\tan 78^\circ=4.705\end{cases}$$

兩式相減得 $\frac{5}{x}=0.44\Rightarrow \frac{h-20}{x}=\frac{h}{x}-\frac{5}{x}\times 4=5.145-0.44\times 4=3.385\approx \tan 74^\circ$

$\therefore$ 俯角約為 74 度．