

一、單選題 (2 題 每題 10 分 共 20 分)

() 1. 下列選項當中何者的值最大? (1) $\sin 20^\circ \cos 20^\circ$ (2) $\sin 35^\circ \cos 35^\circ$
(3) $\sin 50^\circ \cos 50^\circ$ (4) $\sin 65^\circ \cos 65^\circ$ (5) $\sin 80^\circ \cos 80^\circ$.

() 2. 設 $\tan \alpha = \frac{1}{9}$, $\tan(\alpha + \beta) = 1$, 則 $\tan \beta$ 的值為 (1) $\frac{2}{3}$ (2) $\frac{3}{4}$ (3) $\frac{4}{5}$ (4) $\frac{5}{6}$ (5) $\frac{6}{7}$.

二、多選題 (1 題 每題 10 分 共 10 分)

() 1. 已知 $\sin \theta = \frac{2}{3}$ 且 $90^\circ < \theta < 180^\circ$, 則下列何者為真? (1) $\cos \theta = \frac{5}{3}$ (2) $\tan \theta = \frac{\sqrt{5}}{2}$
(3) $\sin 2\theta = -\frac{4\sqrt{5}}{9}$ (4) $\cos 2\theta = \frac{1}{9}$ (5) $\tan 2\theta = 4\sqrt{5}$.

三、填充題 (6 格 每格 10 分 共 60 分)

1. 設 α 為第三象限角, β 為第四象限角, 且 $\sin \alpha = -\frac{4}{5}$, $\cos \beta = \frac{5}{13}$, 求

(1) $\sin(\alpha + \beta) =$ _____ .

(2) $\cos(\alpha - \beta) =$ _____ .

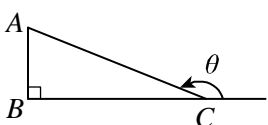
2. 已知 $\sin \theta + \cos \theta = \frac{2}{3}$, 求 $\sin 2\theta =$ _____ .

3. 已知 $\cos \theta = -\frac{3}{5}$ 且 $90^\circ < \theta < 180^\circ$, 求

(1) $\sin \frac{\theta}{2} =$ _____ . (2) $\cos \frac{\theta}{2} =$ _____ . (3) $\tan \frac{\theta}{2} =$ _____ .

四、計算題 (1 題 每題 10 分 共 10 分)

1. 如圖, θ 為一有向角, $\overline{AB} = 2$, $\overline{BC} = 5$, 求 $\sin 2\theta$ 的值.



答 案

一、單選題(2題 每題 10分 共 20分)

1. 3 2. 3

二、多選題(1題 每題 10分 共 10分)

1. 34

三、填充題(6格 每格 10分 共 60分)

1. (1) $\frac{16}{65}$; (2) $\frac{33}{65}$ 2. $-\frac{5}{9}$ 3. (1) $\frac{2\sqrt{5}}{5}$; (2) $\frac{\sqrt{5}}{5}$; (3) 2

四、計算題(1題 每題 10分 共 10分)

1. $-\frac{20}{29}$

解 析

一、單選題(2題 每題 10分 共 20分)

1. 利用 $\sin 2\theta = 2\sin\theta \cos\theta$, 得

$$(1) \sin 20^\circ \cos 20^\circ = \frac{1}{2} \sin 40^\circ. \quad (2) \sin 35^\circ \cos 35^\circ = \frac{1}{2} \sin 70^\circ.$$

$$(3) \sin 50^\circ \cos 50^\circ = \frac{1}{2} \sin 100^\circ = \frac{1}{2} \sin 80^\circ. \quad (4) \sin 65^\circ \cos 65^\circ = \frac{1}{2} \sin 130^\circ = \frac{1}{2} \sin 50^\circ.$$

$$(5) \sin 80^\circ \cos 80^\circ = \frac{1}{2} \sin 160^\circ = \frac{1}{2} \sin 20^\circ.$$

因為 $\sin 80^\circ > \sin 70^\circ > \sin 50^\circ > \sin 40^\circ > \sin 20^\circ$, 故選(3).

$$2. \tan(\alpha + \beta) = \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \cdot \tan \beta} = 1 \Rightarrow \frac{\frac{1}{9} + \tan \beta}{1 - \frac{1}{9} \tan \beta} = 1 \Rightarrow \tan \beta = \frac{4}{5}, \text{ 故選(3).}$$

二、多選題 (1題 每題 10分 共 10分)

1. $\because \sin \theta = \frac{2}{3}$ 且 $90^\circ < \theta < 180^\circ \therefore \cos \theta = -\frac{\sqrt{5}}{3}$.

$$(1) \times: \cos \theta = -\frac{\sqrt{5}}{3}. \quad (2) \times: \tan \theta = -\frac{2}{\sqrt{5}} = -\frac{2\sqrt{5}}{5}.$$

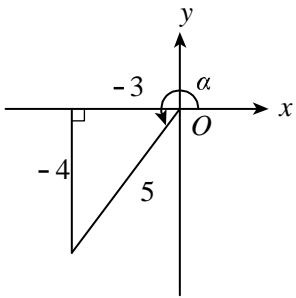
$$(3) \circ: \sin 2\theta = 2\sin \theta \cdot \cos \theta = 2\left(\frac{2}{3}\right)\left(-\frac{\sqrt{5}}{3}\right) = -\frac{4\sqrt{5}}{9}. \quad (4) \circ: \cos 2\theta = 1 - 2\sin^2 \theta = 1 - 2\left(\frac{2}{3}\right)^2 = \frac{1}{9}.$$

$$(5) \times: \tan 2\theta = \frac{\sin 2\theta}{\cos 2\theta} = \frac{-\frac{4\sqrt{5}}{9}}{\frac{1}{9}} = -4\sqrt{5}.$$

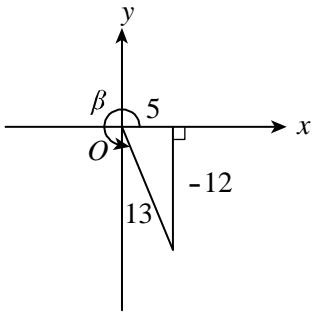
故選(3)(4).

三、填充題(6格 每格 10分 共 60分)

$$1. \sin \alpha = -\frac{4}{5} \Rightarrow \cos \alpha = -\frac{3}{5}$$



$$\cos \beta = \frac{5}{13} \Rightarrow \sin \beta = -\frac{12}{13}$$



$$(1) \sin(\alpha + \beta) = \sin \alpha \cdot \cos \beta + \cos \alpha \cdot \sin \beta = \left(-\frac{4}{5}\right)\left(\frac{5}{13}\right) + \left(-\frac{3}{5}\right)\left(-\frac{12}{13}\right) = \frac{16}{65} .$$

$$(2) \cos(\alpha - \beta) = \cos \alpha \cdot \cos \beta + \sin \alpha \cdot \sin \beta = \left(-\frac{3}{5}\right)\left(\frac{5}{13}\right) + \left(-\frac{4}{5}\right)\left(-\frac{12}{13}\right) = \frac{33}{65} .$$

$$2. \sin \theta + \cos \theta = \frac{2}{3}$$

$$\text{平方} \Rightarrow \sin^2 \theta + \cos^2 \theta + 2 \sin \theta \cdot \cos \theta = \frac{4}{9} \Rightarrow 1 + \sin 2\theta = \frac{4}{9}$$

$$\therefore \sin 2\theta = -\frac{5}{9} .$$

$$3. 90^\circ < \theta < 180^\circ, \cos \theta = -\frac{3}{5} \Rightarrow \sin \theta = \frac{4}{5}$$

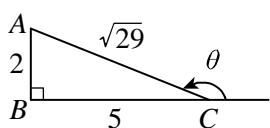
$$45^\circ < \frac{\theta}{2} < 90^\circ, \sin \frac{\theta}{2} > 0 \text{ 且 } \cos \frac{\theta}{2} > 0$$

$$(1) \sin \frac{\theta}{2} = \sqrt{\frac{1 - \cos \theta}{2}} = \sqrt{\frac{1 + \frac{3}{5}}{2}} = \sqrt{\frac{4}{5}} = \frac{2\sqrt{5}}{5} . \quad (2) \cos \frac{\theta}{2} = \sqrt{\frac{1 + \cos \theta}{2}} = \sqrt{\frac{1 - \frac{3}{5}}{2}} = \sqrt{\frac{1}{5}} = \frac{\sqrt{5}}{5} .$$

$$(3) \tan \frac{\theta}{2} = \frac{\sin \frac{\theta}{2}}{\cos \frac{\theta}{2}} = \frac{\sqrt{\frac{4}{5}}}{\sqrt{\frac{1}{5}}} = 2 .$$

四、計算題(1題 每題 10分 共 10分)

1.



$$\overline{AC} = \sqrt{2^2 + 5^2} = \sqrt{29}$$

$$\sin \theta = \sin(180^\circ - \angle ACB) = \sin \angle ACB = \frac{2}{\sqrt{29}} \Rightarrow \cos \theta = -\frac{5}{\sqrt{29}}$$

$$\sin 2\theta = 2 \sin \theta \cdot \cos \theta = 2\left(\frac{2}{\sqrt{29}}\right)\left(-\frac{5}{\sqrt{29}}\right) = -\frac{20}{29} .$$