

一、單一選擇題：每題 10 分，共 50 分

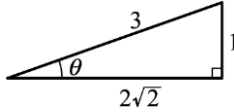
1. 答案：(A)

解析： $\cos^2 \alpha = (\cos x \sin \gamma)^2 = \cos^2 x \sin^2 \gamma$
 $\cos^2 \beta = (\sin x \sin \gamma)^2 = \sin^2 x \sin^2 \gamma$
 $\Rightarrow \cos^2 \alpha + \cos^2 \beta = \cos^2 x \sin^2 \gamma + \sin^2 x \sin^2 \gamma$
 $= \sin^2 \gamma (\cos^2 x + \sin^2 x)$
 $= \sin^2 \gamma$
 $\Rightarrow (1 - \sin^2 \alpha) + (1 - \sin^2 \beta) = \sin^2 \gamma$
 $\Rightarrow 2 = \sin^2 \alpha + \sin^2 \beta + \sin^2 \gamma$
 故選(A)

2. 答案：(E)

解析：在 $\triangle ABC$ 中， $\frac{\overline{AB}}{\sin 45^\circ} = c \dots \dots \dots \textcircled{1}$
 在 $\triangle ABD$ 中， $\angle BDA = 135^\circ \Rightarrow \frac{\overline{AB}}{\sin 135^\circ} = d \dots \dots \dots$
 $\dots \dots \textcircled{2}$
 在 $\triangle ABE$ 中，令 $\angle AEB = \theta$
 $\Rightarrow \theta = \angle EAC + \angle ECA > 45^\circ$
 $\Rightarrow \frac{\overline{AB}}{\sin \theta} = e \dots \dots \dots \textcircled{3}$
 由 $\textcircled{1}$ 、 $\textcircled{2}$ 得 $c = d$ ，又 $\because \theta > 45^\circ \therefore \sin \theta > \sin 45^\circ$
 $\Rightarrow \frac{\overline{AB}}{\sin \theta} < \frac{\overline{AB}}{\sin 45^\circ}$
 由 $\textcircled{1}$ 、 $\textcircled{3}$ 得 $c > e \Rightarrow d = c > e$
 故選(E)

3. 答案：(D)

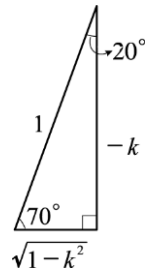
解析：
 $\tan(-540^\circ + \theta)$
 $= \tan(-540^\circ + \theta + 720^\circ)$
 $= \tan(180^\circ + \theta) = \tan \theta$
 又 $\sin \theta = \frac{1}{3} \Rightarrow \tan \theta = \frac{1}{2\sqrt{2}} = \frac{\sqrt{2}}{4}$
 $\therefore \tan(-540^\circ + \theta) = \tan \theta = \frac{\sqrt{2}}{4}$
 故選(D)

4. 答案：(C)

解析： $(a+b+c)(b+c-a) - 3bc = 0$
 $\Rightarrow [(b+c) + a][(b+c) - a] - 3bc = 0$
 $\Rightarrow (b+c)^2 - a^2 = 3bc$
 $\Rightarrow b^2 + 2bc + c^2 - a^2 = 3bc \Rightarrow b^2 + c^2 - a^2 = bc$
 $\cos A = \frac{b^2 + c^2 - a^2}{2bc} = \frac{bc}{2bc} = \frac{1}{2} \Rightarrow \angle A = 60^\circ$
 故選(C)

5. 答案：(E)

解析： $k = \sin(-70^\circ) = -\sin 70^\circ \Rightarrow \sin 70^\circ = -k$ ，作圖



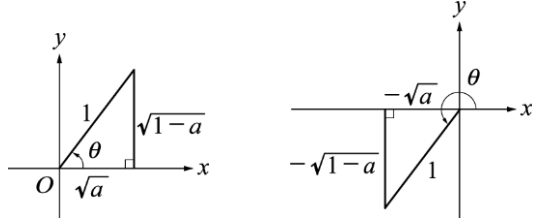
得 $\cos 70^\circ = \sqrt{1-k^2}$ ， $\tan 70^\circ = \frac{-k}{\sqrt{1-k^2}}$ ，
 $\tan(-1280^\circ) = \tan 160^\circ = -\tan 20^\circ = -\frac{\sqrt{1-k^2}}{-k} = \frac{\sqrt{1-k^2}}{k}$
 故選(E)

二、填充題：每題 10 分，共 50 分

1. 答案： $\frac{35}{16}$

解析： $s = \frac{a+b+c}{2} = \frac{5+6+7}{2} = 9$
 $\triangle ABC = \sqrt{s(s-a)(s-b)(s-c)} = \frac{abc}{4R} = r \cdot s$
 $\Rightarrow \sqrt{9 \cdot (9-5) \cdot (9-6) \cdot (9-7)} = \frac{5 \cdot 6 \cdot 7}{4R} = r \cdot 9$
 $\Rightarrow 6\sqrt{6} = \frac{210}{4R} = 9r \Rightarrow R = \frac{210}{24\sqrt{6}}$ ， $r = \frac{6\sqrt{6}}{9}$
 $\therefore \frac{R}{r} = \frac{210}{24\sqrt{6}} \cdot \frac{9}{6\sqrt{6}} = \frac{210 \cdot 9}{24 \cdot 6 \cdot 6} = \frac{210 \cdot 9}{24 \cdot 6 \cdot 6} = \frac{35}{16}$

2. 答案：-2

解析： $\because 0 < a < 1 \therefore \tan \theta = \sqrt{\frac{1-a}{a}} = \frac{\sqrt{1-a}}{\sqrt{a}}$

 $\Rightarrow \begin{cases} \sin \theta = \sqrt{1-a} \\ \cos \theta = \sqrt{a} \end{cases} \text{ 或 } \begin{cases} \sin \theta = -\sqrt{1-a} \\ \cos \theta = -\sqrt{a} \end{cases}$
 $\frac{\sin^2 \theta}{a + \cos \theta} + \frac{\sin^2 \theta}{a - \cos \theta} = \frac{\sin^2 \theta (a - \cos \theta) + \sin^2 \theta (a + \cos \theta)}{(a + \cos \theta)(a - \cos \theta)}$
 $= \frac{2a \sin^2 \theta}{a^2 - \cos^2 \theta} = \frac{2a(1-a)}{a^2 - a} = \frac{2a(1-a)}{a(a-1)} = -2$

3. 答案：(1) $\frac{\sqrt{2}}{2}$ ；(2) $-\frac{\sqrt{3}}{2}$ ；(3) $-\frac{\sqrt{3}}{3}$

解析：(1) $\sin(-315^\circ) = \sin(360^\circ - 315^\circ) = \sin 45^\circ = \frac{\sqrt{2}}{2}$
 (2) $\cos 210^\circ = \cos(180^\circ + 30^\circ) = -\cos 30^\circ = -\frac{\sqrt{3}}{2}$
 (3) $\tan 150^\circ = \tan(180^\circ - 30^\circ) = -\tan 30^\circ = -\frac{\sqrt{3}}{3}$

4. 答案：2

解析： $(\sin \theta + \cos \theta)^2 + (\sin \theta - \cos \theta)^2$
 $= (\sin^2 \theta + 2 \sin \theta \cos \theta + \cos^2 \theta) + (\sin^2 \theta - 2 \sin \theta \cos \theta + \cos^2 \theta)$
 $= 2(\sin^2 \theta + \cos^2 \theta) = 2$

5. 答案： $\frac{11}{2}$

解析： a, b, c 成等差數列 $\Rightarrow a + c = 2b$

設外接圓半徑 R

$$\because \sin(A + C) = \sin(180^\circ - B) = \sin B$$

$$\therefore \text{原式為 } \frac{3(\sin A + \sin C) + 5 \sin B}{2 \sin B}$$

$$= \frac{3\left(\frac{a}{2R} + \frac{c}{2R}\right) + 5 \cdot \frac{b}{2R}}{2 \cdot \frac{b}{2R}}$$

$$= \frac{3(a + c) + 5b}{2b} = \frac{6b + 5b}{2b} = \frac{11}{2}$$