

一、填充題

1. 答案：(1) $\frac{-1+\sqrt{5}}{2}$; (2) 1 ; (3) $4-\sqrt{5}$

解析：(1) $\sin^2 \theta + \sin \theta - 1 = 0 \Rightarrow \sin \theta = \frac{-1 \pm \sqrt{5}}{2}$ (負不合)

$$\therefore \sin \theta = \frac{-1+\sqrt{5}}{2}$$

(2) $\therefore \sin \theta = 1 - \sin^2 \theta \Rightarrow \sin \theta = \cos^2 \theta$

$$\therefore \cos^2 \theta + \cos^4 \theta = \sin \theta + \sin^2 \theta = 1$$

(3) $\sin \theta + \cos^2 \theta + \cos^4 \theta + \cos^8 \theta$

$$\begin{aligned} &= \sin \theta + (\sin \theta + \sin^2 \theta) + \sin^4 \theta \\ &= \sin \theta + 1 + \sin^4 \theta = \sin \theta + 1 + (1 - \sin \theta)^2 \\ &= \sin \theta + 1 + 1 - 2 \sin \theta + \sin^2 \theta \\ &= \sin \theta + 1 + 1 - 2 \sin \theta + 1 - \sin \theta \\ &= 3 - 2 \sin \theta = 3 - 2 \times \left(\frac{-1+\sqrt{5}}{2} \right) = 4 - \sqrt{5} \end{aligned}$$

2. 答案： $1 + \sqrt{5}$

解析：(1) 因 $\sin \theta = \cos^2 \theta > 0$ ，故 $\sin \theta > 0$

$$\sin \theta = \cos^2 \theta \Rightarrow \sin \theta = 1 - \sin^2 \theta$$

$$\Rightarrow \sin^2 \theta + \sin \theta - 1 = 0$$

$$\Rightarrow \sin \theta = \frac{-1 \pm \sqrt{5}}{2} \text{ (取正 } \because \sin \theta = \cos^2 \theta > 0 \text{)}$$

)

$$\text{故 } \sin \theta = \frac{-1+\sqrt{5}}{2}$$

$$\begin{aligned} (2) \frac{1}{1-\sin \theta} + \frac{1}{1+\sin \theta} &= \frac{(1+\sin \theta) + (1-\sin \theta)}{1-\sin^2 \theta} \\ &= \frac{2}{\cos^2 \theta} = \frac{2}{\sin \theta} = \frac{2}{\frac{-1+\sqrt{5}}{2}} = 1 + \sqrt{5} \end{aligned}$$

3. 答案： $\frac{-1+\sqrt{5}}{2}$

解析： $\cos \alpha = \tan \alpha$

$$\Rightarrow \cos \alpha = \frac{\sin \alpha}{\cos \alpha} \Rightarrow \cos^2 \alpha = \sin \alpha$$

$$\Rightarrow 1 - \sin^2 \alpha = \sin \alpha \Rightarrow \sin^2 \alpha + \sin \alpha - 1 = 0$$

$$\Rightarrow \sin \alpha = \frac{-1 \pm \sqrt{5}}{2}$$

$$\because 0^\circ < \alpha < 90^\circ \therefore 0 < \sin \alpha < 1$$

$$\Rightarrow \sin \alpha = \frac{-1+\sqrt{5}}{2}$$

4. 答案： $\sqrt{5} + 1$

解析： $\cos \theta = \tan \theta = \frac{\sin \theta}{\cos \theta} \Rightarrow \sin \theta = \cos^2 \theta = 1 - \sin^2 \theta \Rightarrow$

$$\sin^2 \theta + \sin \theta - 1 = 0$$

$$\Rightarrow \sin \theta = \frac{-1 \pm \sqrt{5}}{2} \text{ (負不合)} \therefore \sin \theta = \frac{\sqrt{5}-1}{2}$$

$$\text{則 } \frac{\sin \theta}{1+\cos \theta} + \frac{1+\cos \theta}{\sin \theta} = \frac{\sin^2 \theta + (1+\cos \theta)^2}{\sin \theta (1+\cos \theta)} =$$

$$\frac{\sin^2 \theta + 1 + 2 \cos \theta + \cos^2 \theta}{\sin \theta (1+\cos \theta)} = \frac{2(1+\cos \theta)}{\sin \theta (1+\cos \theta)}$$

$$= \frac{2}{\sin \theta} = 2 \times \frac{2}{\sqrt{5}-1} = \sqrt{5} + 1$$

5. 答案： $\frac{4}{3}$

解析：由根與係數之關係

$$\begin{cases} \sin \theta + \cos \theta = \frac{-p}{2} \\ \sin \theta \cos \theta = \frac{q}{2} \end{cases}$$

$$\text{又 } \sin \theta - \cos \theta = \frac{1}{\sqrt{3}}$$

$$\Rightarrow (\sin \theta - \cos \theta)^2 = \frac{1}{3}$$

$$\Rightarrow (\sin \theta + \cos \theta)^2 - 4 \sin \theta \cos \theta = \frac{1}{3}$$

$$\Rightarrow \left(\frac{-p}{2} \right)^2 - 4 \cdot \frac{q}{2} = \frac{1}{3}$$

$$\Rightarrow \frac{p^2}{4} - 2q = \frac{1}{3} \Rightarrow p^2 - 8q = \frac{4}{3}$$

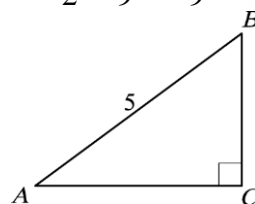
6. 答案： $\frac{50}{9}$

解析： $\sin A - \cos A = \frac{1}{3} \Rightarrow (\sin A - \cos A)^2 = 1 - 2 \sin A \cos A$

$$= \frac{1}{9} \Rightarrow \sin A \cos A = \frac{4}{9}$$

$$\text{又 } \triangle ABC \text{ 面積} = \frac{1}{2} \overline{AC} \times \overline{BC} = \frac{1}{2} (5 \cos A) (5 \sin A)$$

$$= \frac{25}{2} \sin A \cos A = \frac{25}{2} \times \frac{4}{9} = \frac{50}{9}$$



7. 答案： $\frac{5}{12}$

解析：由題目圖知 $\tan \theta = \frac{\overline{OP}}{\overline{PT}} = \frac{1}{\overline{PT}} = \frac{12}{5}$

$$\therefore \overline{PT} = \frac{5}{12}$$

8. 答案：<

解析： $\cos 40^\circ = \sin 50^\circ$

$$\text{又 } \sin 40^\circ < \sin 50^\circ$$

$$\text{故 } \sin 40^\circ < \cos 40^\circ$$

9. 答案： $c > b > a$

解析： $\because a = \frac{1}{2} = \sin 30^\circ$ ， $b = \cos 59^\circ = \sin 31^\circ$ ， $c = \sin 61^\circ$

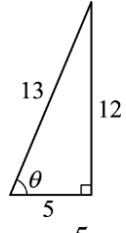
$$\therefore c > b > a$$

10. 答案： $\frac{17}{13}$

解析： $5 \sin \theta - 12 \cos \theta = 0 \Rightarrow 5 \sin \theta = 12 \cos \theta$

$$\Rightarrow \frac{\sin \theta}{\cos \theta} = \frac{12}{5} \Rightarrow \tan \theta = \frac{12}{5}$$

設直角三角形如圖所示



$$\therefore \sin \theta = \frac{12}{13}, \cos \theta = \frac{5}{13}$$

$$\text{故 } \sin \theta + \cos \theta = \frac{12}{13} + \frac{5}{13} = \frac{17}{13}$$