

1. $\triangle ABC$ 的三邊長為 $\overline{AB} = 4$, $\overline{BC} = 3$, $\overline{AC} = 2$, 求 $\overrightarrow{AB} \cdot \overrightarrow{BC} = \underline{-\frac{21}{2}}$ 。(10分)

$$\begin{aligned} \text{解: } \overrightarrow{AB} \cdot \overrightarrow{BC} &= -\overrightarrow{BA} \cdot \overrightarrow{BC} \\ &= -|\overrightarrow{BA}| |\overrightarrow{BC}| \cos B \\ &= -4 \cdot 3 \cdot \frac{4^2 + 3^2 - 2^2}{2 \cdot 4 \cdot 3} \\ &= -\frac{21}{2} \end{aligned}$$

2. $|\vec{u}| = 3$, $|\vec{v}| = 4$, 又 \vec{u} , \vec{v} 的夾角為 120° , 則 $|\vec{u} - \vec{v}| = \underline{\sqrt{37}}$ 。(10分)

$$\begin{aligned} \text{解: } |\vec{u} - \vec{v}|^2 &= |\vec{u}|^2 - 2\vec{u} \cdot \vec{v} + |\vec{v}|^2 = 3^2 - 2(3 \times 4 \times \cos 120^\circ) + 4^2 = 37 \\ \therefore |\vec{u} - \vec{v}| &= \sqrt{37} \end{aligned}$$

3. 設向量 \vec{a} 與 \vec{b} 的夾角為 30° , 且 $|\vec{a}| = 4$, $|\vec{b}| = \sqrt{3}$, 若向量 \vec{b} 與 $2\vec{a} + r\vec{b}$ 垂直, 則實數 $r = \underline{-4}$ 。(10分)

$$\begin{aligned} \text{解: } \because \vec{b} \text{ 與 } 2\vec{a} + r\vec{b} \text{ 垂直} \\ \therefore \vec{b} \cdot (2\vec{a} + r\vec{b}) &= 2\vec{a} \cdot \vec{b} + r|\vec{b}|^2 = 0 \\ \text{又 } \vec{a} \cdot \vec{b} &= |\vec{a}| |\vec{b}| \cos 30^\circ = 4 \times \sqrt{3} \times \frac{\sqrt{3}}{2} = 6 \\ \therefore 2 \times 6 + r \times (\sqrt{3})^2 &= 0 \Rightarrow 3r + 12 = 0 \\ \therefore r &= -4 \end{aligned}$$

4. 坐標平面上已知 $A(3, 0)$, $B(4, 3)$, $C(9, 3)$, 則 \overrightarrow{AC} 在 \overrightarrow{AB} 方向上之正射影為 $\underline{(\frac{3}{2}, \frac{9}{2})}$ 。(10分)

$$\begin{aligned} \text{解: } \overrightarrow{AB} &= (1, 3), \overrightarrow{AC} = (6, 3), \\ \left(\frac{\overrightarrow{AC} \cdot \overrightarrow{AB}}{|\overrightarrow{AB}|^2} \right) \overrightarrow{AB} &= \frac{6+9}{10} (1, 3) = \left(\frac{3}{2}, \frac{9}{2} \right) \end{aligned}$$