

1. (1) 求  $\sin 230^\circ \cos 110^\circ - \cos 230^\circ \sin 110^\circ$  的值为  $\frac{\sqrt{3}}{2}$ 。(9分)

(2) 求  $\cos 70^\circ \cos 40^\circ + \cos 20^\circ \cos 50^\circ$  的值为  $\frac{\sqrt{3}}{2}$ 。(9分)

解：(1) 原式  $= \sin(230^\circ - 110^\circ) = \sin 120^\circ = \frac{\sqrt{3}}{2}$

(2) 原式  $= \cos 70^\circ \cos 40^\circ + \sin 70^\circ \sin 40^\circ = \cos(70^\circ - 40^\circ) = \cos 30^\circ = \frac{\sqrt{3}}{2}$

2. 已知  $\alpha$  為第三象限角且  $\sin \alpha = -\frac{3}{5}$ ， $\beta$  為第四象限角且  $\cos \beta = \frac{1}{2}$ ，試求：

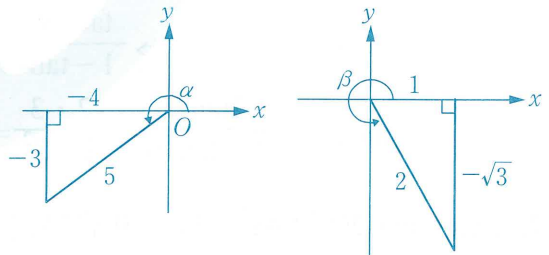
(1)  $\sin(\alpha - \beta) = \frac{-3 - 4\sqrt{3}}{10}$ 。(8分)

(2)  $\cos(\alpha + \beta) = \frac{-4 - 3\sqrt{3}}{10}$ 。(8分)

解：由右圖知  $\sin \alpha = -\frac{3}{5}$ ， $\cos \alpha = -\frac{4}{5}$ ，

$$\sin \beta = -\frac{\sqrt{3}}{2}, \cos \beta = \frac{1}{2}$$

$$\begin{aligned} (1) \sin(\alpha - \beta) &= \sin \alpha \cos \beta - \cos \alpha \sin \beta \\ &= \frac{-3}{5} \times \frac{1}{2} - \frac{-4}{5} \times \frac{-\sqrt{3}}{2} \\ &= \frac{-3 - 4\sqrt{3}}{10} \end{aligned}$$



$$(2) \cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta = \frac{-4}{5} \times \frac{1}{2} - \frac{-3}{5} \times \frac{-\sqrt{3}}{2} = \frac{-4 - 3\sqrt{3}}{10}$$

3. 設  $\tan \alpha = \frac{1}{2}$ ， $\tan(\alpha + \beta) = 1$ ，則  $\tan \beta = \frac{1}{3}$ 。(10分)

解： $\beta = (\alpha + \beta) - \alpha$

$$\Rightarrow \tan \beta = \tan[(\alpha + \beta) - \alpha] = \frac{\tan(\alpha + \beta) - \tan \alpha}{1 + \tan(\alpha + \beta) \tan \alpha} = \frac{1 - \frac{1}{2}}{1 + 1 \times \frac{1}{2}} = \frac{1}{3}$$