

1.  $\triangle ABC$  中,  $(b+c):(c+a):(a+b)=9:11:10$ , 則  $\sin A:\sin B:\sin C=$   $6:4:5$ 。

解: 令  $b+c=9t, c+a=11t, a+b=10t, t>0$

(10分)

$$\Rightarrow 2(a+b+c)=30t$$

$$\Rightarrow a+b+c=15t$$

$$\Rightarrow a=6t, b=4t, c=5t \Rightarrow a:b:c=6:4:5=\sin A:\sin B:\sin C$$

2. 設  $\triangle ABC$  中, 已知  $\angle BAC=60^\circ$ ,  $\overline{AB}=8$ ,  $\overline{AC}=6$ ,

$\angle BAC$  的平分線交  $\overline{BC}$  於  $D$  點, 則  $\overline{AD}=$   $\frac{24\sqrt{3}}{7}$ 。(10分)

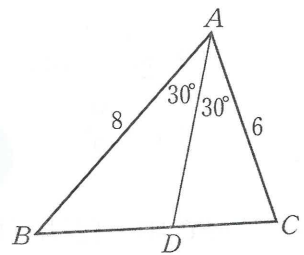
解: 設  $\overline{AD}=x$

$\therefore \triangle ABC$  面積 =  $\triangle ABD$  面積 +  $\triangle ACD$  面積

$$\therefore \frac{1}{2} \times 8 \times 6 \times \sin 60^\circ = \frac{1}{2} \times 8 \times x \times \sin 30^\circ + \frac{1}{2} \times 6 \times x \times \sin 30^\circ$$

$$\Rightarrow 12\sqrt{3} = 2x + \frac{3}{2}x \Rightarrow \frac{7}{2}x = 12\sqrt{3}$$

$$\therefore x = \frac{24\sqrt{3}}{7}$$



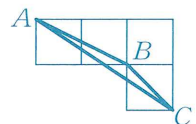
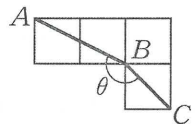
3. 右圖中每一小方格皆為正方形, 求  $\cos \theta$  之值為  $-\frac{3\sqrt{10}}{10}$ 。(10分)

解: 連  $\overline{AC}$ ,

$$\text{則 } \overline{AC} = \sqrt{2^2+3^2} = \sqrt{13}, \text{ 又 } \overline{AB} = \sqrt{1^2+2^2} = \sqrt{5},$$

$$\overline{BC} = \sqrt{1^2+1^2} = \sqrt{2}$$

$$\begin{aligned} \Rightarrow \cos \theta &= \frac{\overline{AB}^2 + \overline{BC}^2 - \overline{AC}^2}{2 \cdot \overline{AB} \cdot \overline{BC}} = \frac{(\sqrt{5})^2 + (\sqrt{2})^2 - (\sqrt{13})^2}{2 \cdot \sqrt{5} \cdot \sqrt{2}} \\ &= \frac{5+2-13}{2 \cdot \sqrt{5} \cdot \sqrt{2}} = \frac{-6}{2\sqrt{10}} = -\frac{3\sqrt{10}}{10} \end{aligned}$$



4. 設  $A, B$  的極坐標分別為  $[3, 70^\circ], [4, 115^\circ]$ , 而  $O$  為極, 則  $\triangle AOB$  的面積 =  $3\sqrt{2}$ 。(10分)

解:  $\triangle AOB = \frac{1}{2} \overline{OA} \cdot \overline{OB} \times \sin 45^\circ$

$$= \frac{1}{2} \times 3 \times 4 \times \frac{\sqrt{2}}{2}$$

$$= 3\sqrt{2}$$

